3001. Differentiate the area formula $A = \frac{1}{2}h(a + b)$ with respect to t, treating a, b and h as variables each depending on t.

3002. Rewrite the improper algebraic fraction as the sum of a constant and a proper fraction.

3003. Use $m_1 m_2 = -1$ to find the x then y coordinates of the intersections. Substitute these into the second parabola to find k.

3004. Assume, for a contradiction, that three forces act on an object as in the question, and that the object is in equilibrium. Resolve in the y direction to find the directions of the forces. Then take moments and find a contradiction.

3005. Differentiate, either by the quotient rule or using a standard result (and the chain rule). Then sub into the LHS as an expression. Use the third of the Pythagorean trig identities to simplify.

3006. Square both sides, remembering that, in doing so, you may introduce new points. Then use a double-angle formula.

3007. (a) Set the probability to 100%.
(b) Find the sum of an arithmetic series.

3008. Use $\tan\theta = m$, where m is the gradient and θ is the angle of inclination (between a line and the x axis).

3009. The first boundary equation is a straight line. The second boundary equation is a circle.

3010. Two are true, one is false.

3011. Consider the different equations in each quadrant, beginning with $\frac{x}{a} + \frac{y}{b} = 1$.

3012. For f to be invertible, it must be one-to-one. So, sketch the graph of $y = x + \frac{4}{x}$, finding any turning points.

3013. (a) Note that A, B, C have the same z coordinate, so you need only use 2D Pythagoras. Also note that B and C are symmetrical.
(b) Use 3D Pythagoras, and the result from (a).

3014. (a) Reaction is perpendicular to the oblique side of the part. Friction is parallel to it.
(b) The part is symmetrical, so you can ignore the horizontal and consider one half of it. Resolve vertically. Remember that the contact force is the Pythagorean sum of the perpendicular reaction and friction forces.

3015. Show that $y = x^3 - x$ has rotational symmetry around the origin, by comparing the values at x and $-x$. Then translate the problem by $2\mathbf{j}$.

3016. Consider the sin and cos cases separately. For each, find the interval subset of the domain which is successful, and consider the domain $[0, \pi/4)$ as the possibility space.

3017. Substitute to find the values of A and k. Then set up a definite integral.

3018. Take out a factor of x on the LHS and differentiate implicitly with respect to y using the product rule. Substitute $y = -1$ and show that $\frac{dx}{dy} = 0$.

3019. Set the output to y, and rearrange to make x the subject.

3020. When the contact face is changed, scaling the area by $\frac{A_2}{A_1}$, consider the scale factors of

① area in contact,
② total reaction force,
③ pressure.

3021. Use the substitution $u = 2 - \sqrt{x}$ to reach

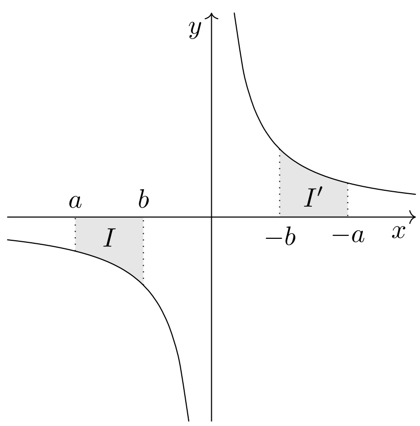
$$\int_2^1 \frac{4-u}{u}(2u-4)\,du.$$

Then multiply out and integrate.

3022. Consider the reflection in $y = -x$ as reflection in $y = x$ followed by a rotation of 180° around the origin.

3023. Consider the possibility space as a list of orders of OOOEEE, and use $p = \frac{\text{successful}}{\text{total}}$.

3024. Compare two signed areas between $y = 1/x$ and the x axis, which are images of each other under rotation 180° around the origin:



3025. Take logs base x of both sides of the equation of the curve. Then use

$$\log_x y \equiv \frac{1}{\log_y x}$$

to find $\log_y x$. Add the terms.

3026. The string can be horizontal, with $\theta = 0$. There are two possibilities for the boundary case with greatest θ. These are ① the sledge lifting off the ground, or ② the sledge not accelerating. Draw a force diagram, and consider these in turn.

3027. (a) Differentiate using the quotient rule.
   (b) Consider x values for which the denominator is undefined, and also the behaviour as $x \to \pm\infty$.
   (c) Use the answers to (a) and (b).
   (d) Consider the graphical behaviour of the N-R method, in which the new approximation to a root is the x intercept of a tangent.

3028. Differentiate by the chain rule, and set the first derivative to zero.

3029. Enact the differential operator (which is implicit differentiation by the product and chain rules), and rearrange to make $\frac{dy}{dx}$ the subject.

3030. (a) Express $\sin x$ as $\cos x \tan x$.
   (b) Solve using the factor theorem. You should get four roots.

3031. (a) The geometric series has first term $a = 1$ and common ratio $r = x$.
   (b) Use the generalised binomial expansion.

3032. (a) Sketch the case with four intersections, and compare it to the diagram given.
   (b) Use differentiation to find the value of k in the diagram given.

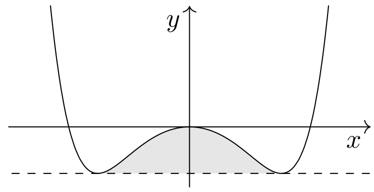
3033. Equate the x coordinates, and solve for t. Show that, at this time, the y coordinates are also equal.

3034. Show that the equation $f(x) - g(x) = 0$ cannot be a quadratic equation, and hence that the leading coefficients of the parabolae must be the same.

3035. These make up the *remainder theorem*, which is a generalisation of the factor theorem. Rewrite the division statement as a multiplication statement, in the manner of the following:

$$13 \div 5 = 2 \text{ remainder } 3,$$
$$13 = 2 \times 5 + 3.$$

3036. Since $y = k$ has gradient zero, find SPs. Find k, then set up a single definite integral for the area between the line and the curve:



3037. In each, find the range of the denominator first.

3038. Since the instruction is "Find...", as opposed to "Determine...", use the definite integration facility on your calculator to find both the true value and the approximation.

3039. Solve simultaneously for x to reach $x^2 = k$, where k is a function of a and b. Justify that $k > 0$. Then show that each of the two x values must produce precisely one y value.

3040. Find two functions which are non-identical, yet for which the order of application doesn't affect the composition.

3041. The statement can be corrected by changing either the central statement, or the critical value k.

3042. Differentiate by the chain rule, or use a symmetry argument.

3043. The fan-belt exerts two forces of magnitude T on each wheel. These are symmetrical in the angle bisector of the vertex. Resolve in this direction.

3044. (a) Find consecutive integers a and $a+1$ such that $f(a) < 0$ and $f(a+1) > 0$. You might sketch $y = x^2$ and $y = 3^{-x}$ to find the approximate location of the root.
   (b) Show that the function f is increasing over the domain.

3045. Calculate the original probability using an (X, Z) possibility space. Having done this, set up an (X, Y) possibility space, and work out what the information $X+Y = 8$ tells you about X. Restrict the (X, Z) possibility space using this information, and recalculate the probability.

3046. (a) Consider the symmetry of the equation. What happens if you switch x and y?
   (b) Differentiate implicitly with respect to x, and set $\frac{dy}{dx} = 0$ to find the points with tangents parallel to the x axis. You can use symmetry to find the points with tangents parallel to the y axis.

3047. Consider the largest domain D of $x \mapsto \ln x$. Then set up an inequality for $x^2 - 1 > ...$, based on the above. Consider the domain of $x \mapsto x^2 - 1$ which will produce D as its range.

    Remember that "largest", in this context, means "containing the most numbers".

3048. (a) Find the acceleration and use *suvat*.
    (b) Set up a new *suvat*, using a new acceleration. You will need a positive initial velocity and a negative acceleration (or vice versa).

3049. Read the question carefully: it is asking you, in rather formal language, to find the tangent line to a semicircle. Sketch the graphs $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$, and then use circle geometry. This is much easier than using calculus.

3050. Draw a clear sketch of two hexagons, together with a circle marking the path of vertices of the inner hexagon. Then use trigonometry to find the ratio of the radii, thus the ratio of areas.

3051. Multiply up by the denominators. Simplify the RHS using a difference of two squares. Expand the terms on the LHS using the binomial expansion. When you end up with a cubic, differentiate to show that the function is increasing everywhere.

3052. Sketch the scenario carefully. Split the region in half at the line $x = \frac{1}{2}k$, and then integrate.

3053. (a) Describe the correlation.
    (b) Read this off the graph.
    (c) Consider the fact that the sample is taken among those who are registered as self-employed for *both* years.

3054. Use the substitution $x = \sin\theta$. You'll then need the double-angle formula $\cos 2\theta \equiv 2\cos^2\theta - 1$ to perform the resulting integral.

3055. "Stationary with respect to x" means that the value of the derivative with respect to x is zero. Express this as an equation using the differential operator $\frac{d}{dx}$, and then enact said operator using the quotient rule.

3056. Statement ② is false.

3057. (a) Find $\frac{dy}{dx}$ and substitute.
    (b) i. Use the product rule.
        ii. Substitute into the original DE, and divide through by $e^{-2x} \neq 0$.
        iii. Integrate the result in ii.

3058. This is a quadratic in $\sin t$.

3059. An even polynomial function has the y axis as a line of symmetry. This means that $y = \mathrm{f}(x)$ only intersects one of these with certainty.

3060. Split each term into partial fractions.

3061. The key question is whether the functions in the right-hand statement are one-to-one or not.

3062. Draw a clear diagram, including the radii of the bowl and the spheres to all relevant points.

3063. For points of inflection, the second derivative must be zero and change sign.

3064. $\mathrm{f}(x)$ is quadratic in $e^x \cos x$. Complete the square to find the vertex.

3065. Let $y = \mathrm{f}(x)$ and $z = \mathrm{g}(x)$. Then express $\frac{dy}{dz}$ in terms of derivatives with respect to x.

3066. Be careful here – the stretch factor k has not been applied to the entire right-hand side.

3067. The statement is false. Find a counterexample.

3068. You can prove this using the reverse chain rule (by integration) or the chain rule (differentiation). For the former, quote the result

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\, dx = \ln|\mathrm{f}(x)| + c.$$

    For the latter, use the chain rule to differentiate $\ln|\sec x| + c$. For a formal proof, consider $\sec x \geq 0$ and $\sec x < 0$ separately. Quote the derivative of sec, or else find it using the chain rule.

3069. (a) Draw a sketch, and consider the effect on the line $P = 6x + 9y$ of increasing P.
    (b) Use either the method given in part (a), or eliminate y and optimise P with respect to x.

3070. Place a pawn in the first row, then the second row, and so on.

3071. Sketch the graph of $y = \cot x$ first, using the fact that $\cot x \equiv \tan\left(\frac{\pi}{2} - x\right)$. Restrict its domain and codomain to the central one-to-one branch. Then reflect this curve in the line $y = x$.

3072. Multiply up by the denominator, then raise both sides to the power 6. At the end, test the only possible value.

3073. The rectangle has unspecified length, so you need only show that one of the perpendicular heights of the triangle is shorter than 6 cm.

3074. (a) Consider the fact that, of the monic cubic curves whose point of inflection is at the origin, only $y = x^3$ has a stationary point of inflection.
(b) Consider a translation of the curve $y = x^3$.

3075. The triangle is equilateral. Find the side length by Pythagoras, and thus the area.

3076. A combinatorial approach is easier. The original possibility space consists of ${}^{10}\text{C}_3 = 120$ outcomes, which is then restricted by the condition given. For the remaining seven bottles to form exactly two distinct groups, the three fallen bottles must form a contiguous three ABC. Work out in how many different ways this can happen.

3077. Set up vertical and horizontal *suvat*s. Eliminate t to find the equation of the trajectory (in terms of u_x). Then equate coefficients and solve a pair of simultaneous equations for k and u_x.

3078. You don't need calculus. Solve simultaneously and show that the resulting equation in one variable has a double root.

— Alternative Method —

Differentiate implicitly with respect to x. Set $\frac{dy}{dx} = -1$. Then substitute the resulting equation back into the equation of the curve and solve.

3079. Write $\cot x \equiv \frac{\cos x}{\sin x}$, and use the quotient rule.

3080. Count the number of successful outcomes.

3081. The result here is essentially the same as if the graphs were $y = x^2 - 1$ and $y = x^4 - 1$.

3082. Multiply tops and bottoms of the main fractions by the denominators of their inlaid fractions.

3083. Consider the equilateral triangle formed by the centres. Find its area, then subtract the areas of the sectors.

3084. Rearrange into the form $\text{f}(x) = \text{g}(y)$.

3085. The key thing is the short length of time for which motion of the tablecloth takes place.

3086. Differentiate implicitly, and set $\frac{dy}{dx}$ to zero.

3087. The common difference can be $\pm 1, \pm 2$: count the successful outcomes case by case.

3088. Factorise and consider the nature of the roots.

3089. Use the small-angle approximation $\cos\theta \approx 1 - \frac{1}{2}\theta^2$.

3090. Use the formula $s^2 = \dfrac{\sum x^2 - n\bar{x}^2}{n}$.

3091. Use the double-angle identity $\cos 2t \equiv 1 - 2\sin^2 t$.

3092. Use the factor theorem.

3093. Find the intersection, and show that the gradients are perpendicular.

3094. Show that there is more than one way of setting up the three forces such that the resultant force has magnitude 8 N.

3095. Successful triangles are equilateral, with each side a face diagonal of length $\sqrt{2}$. For a conditioning approach, choose one vertex wlog. Then consider the probability that the next vertex lies along a face diagonal, and so on.

— Alternative Method —

For a combinatorial approach, the possibility space consists of the ${}^8\text{C}_3$ groups of three vertices which can be chosen. Consider one face $ABCD$. Count up the number of successful outcomes with

- one vertex on face $ABCD$,
- two vertices on face $ABCD$.

3096. (a) Find/write down the first two derivatives of e^x and the approximating quadratic. Then sub zero in and compare values.
(b) Continue in the same vein.

3097. The centres of the circles are at $(k, -k)$. Consider this as the parametric equation of a line. Region R consists of all points within $\sqrt{2}$ of this line.

3098. (a) Set the denominator to zero.
(b) Write the improper fraction as the sum of a constant and a proper fraction. You don't need to use calculus.

3099. Use the formula $\displaystyle\int_{t_1}^{t_2} y \frac{dx}{dt}\, dt$.

3100. Factorise the equation of the graph, and consider the multiplicity of the factors.

— End of 31st Hundred —